Solutions

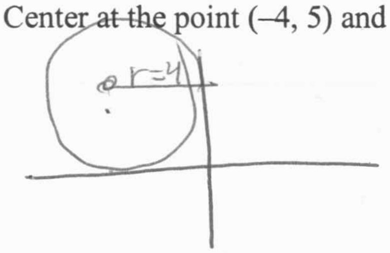
CONICS

Questions 1 – 4: Write the equation of a circle, in standard form, with the given conditions.

1) $r = 5$, $(h, k) = (2, -3)$

$$\boxed{(x-2)^2+(y+3)^2=25}$$

2) Center at the point (–4, 5) and tangent to the y-axis.



$$\boxed{(x+4)^2+(y-5)^2=16}$$

3) Center at the point (4, –7) and containing the point (–2, 6).

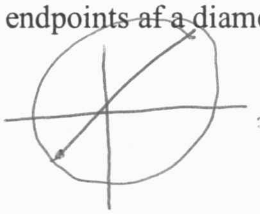
$r=\sqrt{(4+2)^2+(-7-6)^2}$

$=\sqrt{36+169}$

$r=\sqrt{205}$

$$\boxed{(x-4)^2+(y+7)^2=205}$$

4) With endpoints of a diameter at the points (5, 7) and (–2, –2).



midpoint=center

$=\left(\frac{5+-2}{2}, \frac{7+-2}{2}\right)$

$\left(\frac{3}{2}, \frac{5}{2}\right)$

diameter

$=\sqrt{49+81}$

$=\sqrt{130}$

radius $=\frac{\sqrt{130}}{2}$

$$\boxed{(x-1.5)^2+(y-2.5)^2=\frac{130}{4}}$$

Questions 5 – 10: Convert to standard form. Identify the features specified for each.

5) $y^2 - 4y + 4x + 4 = 0$

Vertex: (0,2)

Focus: (-1,2)

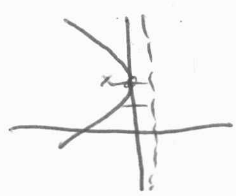
Directrix: x=1

$y^2-4y\ \underline{+4}\ +4x=-4\ \underline{+4}$

$(y-2)^2=-4x$

$4p=4$

$p=1$



6) $2x^2 + 2y^2 - 12x + 8y - 24 = 0$

Center: (3,-2)

Radius: 5

$x^2+y^2-6x+4y-12=0$

$(x^2-6x\ \underline{+9})+(y^2+4y\ \underline{+4})=12\ \underline{+9}\ \underline{+4}$

$(x-3)^2+(y+2)^2=25$

7) $y^2 - 4x^2 - 16x - 2y - 19 = 0$

Center: (-2,1)

$y^2 - 4x^2 - 16x - 2y - 19 = 0$

$y^2 - 2y + \underline{1} - 4(x^2 + 4x \underline{+4}) = 19 \underline{-16+1}$

$\frac{(y-1)^2}{4} - \frac{4(x+2)^2}{4} = \frac{4}{4}$

$$\boxed{\frac{(y-1)^2}{4} - \frac{(x+2)^2}{1} = 1}$$

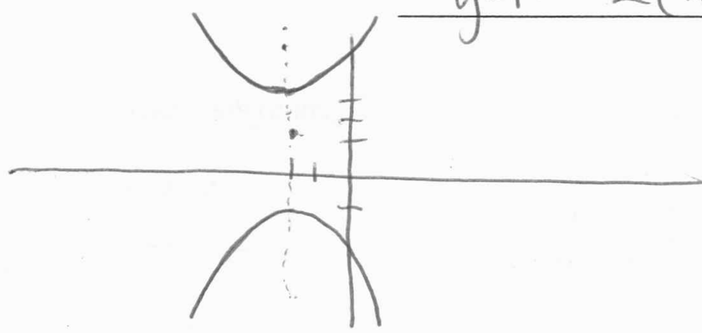
Transverse axis:: $x = -2$ (equation)

Vertices: $(-2,3)\ (-2,-1)$

Foci: $(-2, 1 \pm \sqrt{5})$

Equations of asymptotes:

$y - 1 = \pm 2(x+2)$



$a^2 + b^2 = c^2$

$4 + 1 = c^2$

$\sqrt{5} = c$

8) $2x^2 + 3y^2 - 8x + 6y + 5 = 0$

$2x^2 - 8x \_\_ + 3y^2 + 6y \_\_ = -5$

$2(x^2 - 4x \underline{+4}) + 3(y^2 + 2y \underline{+1}) = -5 \underline{+8} + 3$

$\frac{2(x-2)^2}{6} + \frac{3(y+1)^2}{6} = \frac{6}{6}$

$$\boxed{\frac{(x-2)^2}{3} + \frac{(y+1)^2}{2} = 1}$$

$r_x = \sqrt{3}$ $\quad r_y = \sqrt{2}$

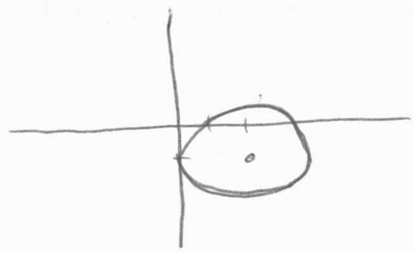
Center: $(2,-1)$

Major axis: $y = -1$ (equation)

Minor axis: $x = 2$ (equation)

Vertices: $(2 \pm \sqrt{3}, -1)$

Foci: $(1,-1)\ (3,-1)$



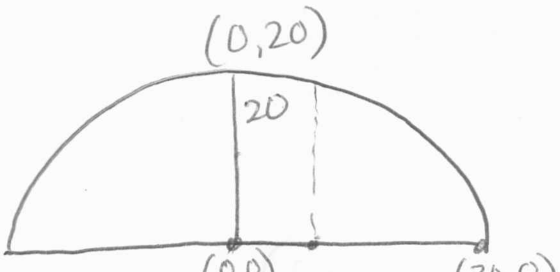
$a^2 - b^2 = c^2$

$3 - 2 = c^2$

$1 = c^2$

$1 = c$

9) An arch in the form of a semi-ellipse is 60 feet wide and 20 feet high at the center. Find the heightof the arch 10 feet from the center.



$\frac{x^2}{30^2} + \frac{y^2}{20^2} = 1$

$\frac{x^2}{900} + \frac{y^2}{400} = 1$

let $x = 10$ ... solve for y

$\frac{100}{900} + \frac{y^2}{400} = 1$

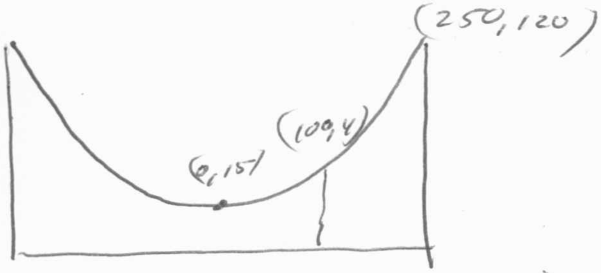
$\sqrt{\frac{y^2}{400}} = \sqrt{\frac{8}{9}}$

$\frac{y}{20} = \frac{\sqrt{8}}{3} \rightarrow 3y = 20\sqrt{8}$

$y = \frac{20\sqrt{8}}{3} = \boxed{\frac{40\sqrt{2}}{3}}$

$\boxed{18.86\text{ft}}$

10) The cables of a suspension bridge are in the shape of a parabola. The towers supporting the cables are 500 feet apart and 120 feet high. If the cables are at a height of 15 feet midway between the towers, what is the height of the cables at a point 100 feet from the center of the bridge?



$x^2 = a(y-15)$

$(250)^2 = a(120-15)$

$62500 = 105a$

$x^2 = \frac{12500}{21}(y-15)$

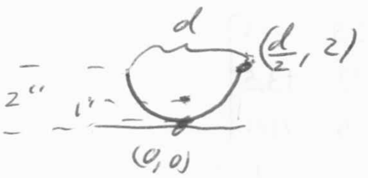
let $x = 100$

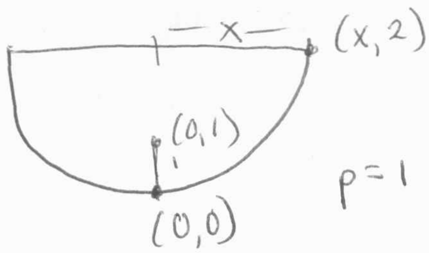
$100^2 = \frac{12500}{21}(y-15)$

$16.8 = y - 15$

$\boxed{31.8 \text{ feet}}$

11) A sealed-beam headlight is in the shape of a paraboloid of revolution. The bulb, which is placed at the focus, is 1 inch from the vertex. If the depth is to be 2 inches, what is the diameter of the headlight at its opening?





$p = 1$

$x^2 = 4p(y)$

$x^2 = 4y$

let $y = 2$

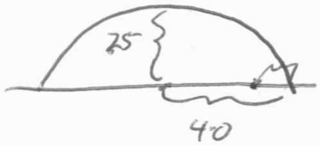
$x^2 = 8$

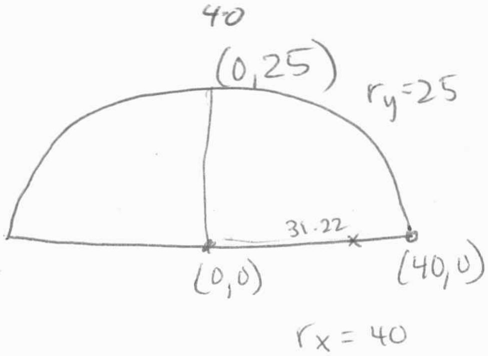
$x = \sqrt{8} = 2\sqrt{2}$

$2(2\sqrt{2})$

$\boxed{4\sqrt{2} \approx 5.66 \text{in}}$

12) A whispering gallery is 80 feet wide. The height of the ceiling in the center is 25 feet. Where are the foci in relation to the walls of the gallery?





$\frac{x^2}{40^2} + \frac{y^2}{25^2} = 1$

$\frac{x^2}{1600} + \frac{y^2}{625} = 1$

$a^2 - b^2 = c^2$

$1600 - 625 = c^2$

$975 = c^2$

$5\sqrt{39} = c$

$31.22 = c$

$40 - 31.22$

$\boxed{8.78 \text{ ft from the walls}}$

CONICS

State the dimensions of the following matrices.

13) $\begin{bmatrix} 3 & -2 & 7 & 9 \\ 1 & 0 & -3 & 5 \\ -8 & 2 & 10 & -6 \end{bmatrix}$

3x4

14) $\begin{bmatrix} 3 & -2 & 7 & 9 \\ 1 & 0 & -3 & 5 \\ -8 & 2 & 10 & -6 \end{bmatrix}$

3x4

15) $\begin{bmatrix} 9 \\ 6 \\ 5 \end{bmatrix}$

3x1

16) $\begin{bmatrix} 6 & 8 & -17 \\ -7 & -5 & 15 \\ 1 & 143 & 2 \\ 11 & 13 & -3 \end{bmatrix}$

4x3

Solve for the variable(s):

17) $\begin{bmatrix} -3 & 5 \\ \boxed{25} & -2 \end{bmatrix} - 3\begin{bmatrix} 0 & -2 \\ x & 4 \end{bmatrix} = \begin{bmatrix} -3 & 11 \\ \boxed{15} & -14 \end{bmatrix}$

$25 - 3x = 15$

$-3x = -10$

$\boxed{x = \frac{10}{3}}$

18) $-5\begin{bmatrix} 5 & 6 \\ 10 & -7 \\ 8 & x \\ \boxed{1} & -6 \\ 7 & 8 \end{bmatrix} + 4\begin{bmatrix} 0 & 1 \\ 1 & -2 \\ 2 & 3 \\ \boxed{4} & 11 \\ -5 & 3 \end{bmatrix} = 2\begin{bmatrix} 12.5 & -13 \\ -23 & 13.5 \\ -16 & 100 \\ \boxed{y} & 37 \\ -27.5 & -14 \end{bmatrix}$

$-5x + 12 = 200$

$-5x = 88$

$\boxed{x = -\frac{88}{5}}$

$-5 + 16 = 2y$

$+11 = 2y$

$\frac{11}{2} = y$

Find the product. If the product is not defined, explain why.

19) $\begin{bmatrix} 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

1x2 ✓

2x1

sorry...

$[15 - 7] = [8]$

20) $\begin{bmatrix} -1 & 0 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 4 & -6 \end{bmatrix}$

2x2

1x2

columns of first matrix isnt equal to # rows of 2nd

21) $\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}$

2x2 ✓

2x2

$\begin{bmatrix} -12 & 15 \\ 8 & -4 \end{bmatrix}$

22) $\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix}$

3x2 ✓

2x2

3x2

$\begin{bmatrix} 11 & 35 \\ 8 & 0 \\ -9 & 7 \end{bmatrix}$

23) $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix}$

2x3 ✓

3x2

2x2

$\begin{bmatrix} -3 & -8 \\ 74 & -50 \end{bmatrix}$

18 + 48 + 8

74

2 − 36 − 16

−52

Solve for the variables.

24) $\begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ y \end{bmatrix}$

25) $\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} y & -4 \\ -13 & 8 \end{bmatrix}$

$-2+x+6=6$
$x+4=6$
$\boxed{x=2}$

$0-2x+12=y$
$0-2(2)+12=y$
$-4+12=y$
$\boxed{8=y}$

$36+2-6=y$
$32=y$

$-18+2x-1=-13$
$-19+2x=-13$
$2x=6$
$x=3$

Questions 26 – 27: Write an inventory matrix and a cost per item matrix. Then use multiplication to write a total cost matrix.

26) A softball team needs to buy 12 bats at \$21 each, 45 balls at \$4 each, and 15 uniforms at \$30 each.

$\cancel{\begin{bmatrix} 12 \\ 45 \\ 15 \end{bmatrix} \begin{bmatrix} 21 & 4 & 30 \end{bmatrix}}$

$\cancel{3\times1}$
$1\times3 = 3\times3$

$\begin{bmatrix} 21 & 4 & 30 \end{bmatrix} \begin{bmatrix} 12 \\ 45 \\ 15 \end{bmatrix} = \$882$

1×3
$3\times1 = [1\times]$

27) A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvasses. For class 2, the teacher buys 20 tubes of paint, 14 brushes, and 15 canvasses. Each tube of paint costs #3.35, each brush costs \$1.75, and each canvass costs \$4.50.

$\cancel{\begin{array}{c|cc} & 1 & 2 \\ \hline P & 24 & 20 \\ B & 12 & 14 \\ C & 17 & 15 \end{array} \quad [\;]}$

$\begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} P \quad B \quad C \\ \begin{bmatrix} 24 & 12 & 17 \\ 20 & 14 & 15 \end{bmatrix} \end{array} \begin{bmatrix} 3.35 \\ 1.75 \\ 4.50 \end{bmatrix} = \begin{bmatrix} 177.9 \\ 159 \end{bmatrix} \begin{array}{l} \text{Class 1} \\ \text{Class 2} \end{array}$

2×3
3×1

Questions 28 – 29: Find the inverse of each matrix, if it exists. No calculator. Inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

28) $\begin{bmatrix} -2 & 6 \\ -1 & 3 \end{bmatrix}$

$\frac{1}{-6-(-6)}$

$\frac{1}{0}$

No inverse exists.
det = 0

29) $\begin{bmatrix} 14 & 8 \\ 6 & 4 \end{bmatrix}$

$\frac{1}{56-48}\begin{bmatrix} 4 & -8 \\ -6 & 14 \end{bmatrix}$

$\frac{1}{8}\begin{bmatrix} 4 & -8 \\ -6 & 14 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{3}{4} & \frac{7}{4} \end{bmatrix}$

Inverse

Questions 30 – 31: Solve for *X*. 2x2 by hand, 3x3 with calculator.

30) $\begin{bmatrix} 5 & 4 \\ -3 & 2 \end{bmatrix} X = \begin{bmatrix} 10 \\ -16 \end{bmatrix}$

$Ax = B$

$A^{-1} = \frac{1}{10+12}\begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix} = \frac{1}{22}\begin{bmatrix} \ \end{bmatrix}$

$x = A^{-1}B$

$x = \frac{1}{22}\begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ -16 \end{bmatrix} = \frac{1}{22}\begin{bmatrix} 84 \\ -50 \end{bmatrix}$

$X = \begin{bmatrix} \frac{42}{11} \\ -\frac{25}{11} \end{bmatrix}$

31) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 4 & 2 \\ 3 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} -5 \\ 15 \\ -7 \end{bmatrix}$

A X B

on calc ...

$A^{-1} \cdot B = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

Questions 32 – 33: Solve the following systems using matrices. Check your answers. All 2x2 systems must be solved by hand. Systems 3x3 and larger may be solved with a calculator. However, show the matrix equation for all systems.

32) $\begin{cases} 3x - 7y = 7 \\ 7x + 3y = 3 \end{cases}$

$\underbrace{\begin{bmatrix} 3 & -7 \\ 7 & 3 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$A^{-1} = \frac{1}{9+48}\begin{bmatrix} 3 & 7 \\ -7 & 3 \end{bmatrix}$

$\frac{1}{57}\begin{bmatrix} \ \end{bmatrix}$

$A^{-1} \cdot B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{57}\begin{bmatrix} 3 & 7 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \frac{1}{57}\begin{bmatrix} 42 \\ -40 \end{bmatrix}$

$x = 14/19$

$y = -57/40$

33) $\begin{cases} x + 5y - 10z = 13 \\ 2x - y + 3z = 18 \\ -4x + 6y + 12z = 7 \end{cases}$

$x = A^{-1}B$

$\underbrace{\begin{bmatrix} 1 & 5 & -10 \\ 2 & -1 & 3 \\ -4 & 6 & 12 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 13 \\ 18 \\ 7 \end{bmatrix}}_{B}$

$x = \frac{503}{58}$

$y = \frac{1133}{290}$

$z = \frac{441}{290}$

34) A flower farmer wants to plant three types of bulbs: gladiolas, irises, and tulips. The gladiolas cost \$75per acre to plant, the irises cost \$100 per acre to plant, and the tulips cost \$50 per acre to plant.

The farmer wants to plant 200 acres of bulbs and spend a total of \$15,000. The farmer decides to plant twice as many gladiolas as irises. Write a system of equations and use a matrix to find the total number of acres of each type of flower.

$\cancel{\begin{bmatrix} G & I & T \end{bmatrix}\begin{bmatrix} 75 \\ 100 \\ 50 \end{bmatrix}}$

1 x 3

3 x 1

G = # gladiolas

I = # irises

T = # tulips

$\begin{cases} G + I + T = 200 \\ 75G + 100I + 50T = 15000 \\ \cancel{2I = G} \\ -G + 2I = 0 \end{cases}$

$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 75 & 100 & 50 \\ -1 & 2 & 0 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} G \\ I \\ T \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 200 \\ 15000 \\ 0 \end{bmatrix}}_{B}$

3x3 · 3x1 = 3x1

$X = A^{-1}B$ (on calc)

$= \begin{bmatrix} 100 \\ 50 \\ 50 \end{bmatrix}$

$\boxed{G = 100 \quad I = 50 \quad T = 50}$

FUNCTIONS

35) The cost of having a carpet installed is \$25.00 for delivery and \$1.50 per square yard for the actual installation.

a) Find a linear equation that models the cost of having a carpet delivered and installed.

$f(x) = 1.50x + 25$

x = square yard

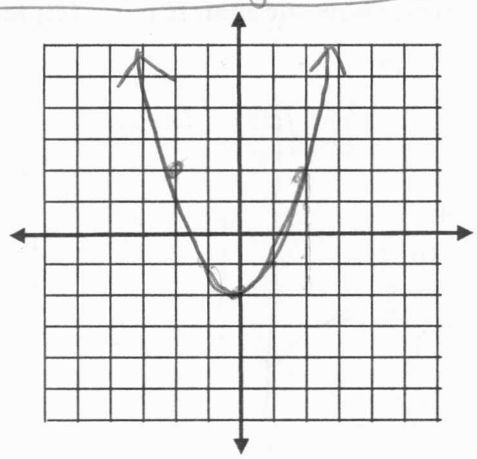
b) Find the number of square yards of carpet installed if the bill for delivery and installation is \$60.25 without the tax.

$60.25 = 1.50x + 25$

$35.25 = 1.50x$

$\boxed{23.5 \text{ square yards}}$

36) Graph: $y = x^2 - 2$



37) Given $f(x) = x^2 - 1$, evaluate each of the following and *simplify*:

a) $f(-2)$

$= (-2)^2 - 1$

$= 4 - 1$

$\boxed{= 3}$

b) $f(a+b)$

$= (a+b)^2 - 1$

$\boxed{= a^2 + 2ab + b^2 - 1}$

c) $f\left(\frac{2}{x}\right)$

$= \left(\frac{2}{x}\right)^2 - 1$

$\boxed{= \frac{4}{x^2} - 1}$

38) Find the domain and range of each function:

a) $g(x) = \sqrt{x - 1.5}$

D: $[1.5, \infty)$

R: $[0, \infty)$

$x - 1.5 \geq 0$

$x \geq 1.5$

b) $h(x) = \dfrac{3}{x^2 - x - 20}$

D: $(-\infty, -4) \cup (-4, 5) \cup (5, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

$(x-5)(x+4)$

$x \neq 5 \quad x \neq -4$

39) Find the inverse of each function. Is the function one-to-one?

a) $f(x) = \frac{\sqrt{x}}{2}$

$x = \frac{\sqrt{y}}{2}$

$2x = \sqrt{y}$

$(2x)^2 = y$

$4x^2 = y$

$\boxed{f^{-1}(x) = 4x^2}$ yes, one-to one

b) $g(x) = \frac{1}{2x-3}$

$x = \frac{1}{2y-3}$

$x(2y-3) = 1$

$2y - 3 = \frac{1}{x}$

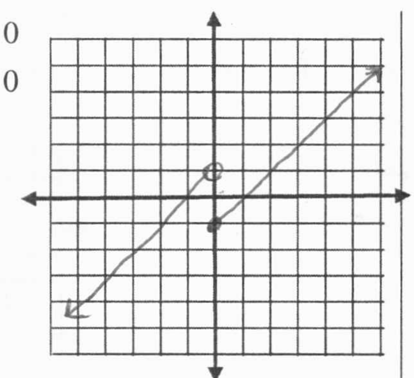
$2y = \frac{1}{x} + 3$

$y = \frac{2}{x} + 6$ or $\frac{2+6x}{x}$

$\boxed{f^{-1}(x) = \frac{2}{x} + 6}$ yes one to one

40) Graph each function and determine its domain and range.

a) $f(x) = \begin{cases} x+1, & x < 0 \\ x-1, & x \geq 0 \end{cases}$



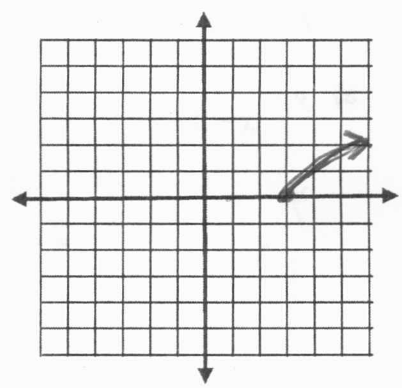
D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

b) $g(x) = \sqrt{x-3}$

$x - 3 \geq 0$

$x \geq 3$



D: $[3, \infty)$

R: $[0, \infty)$

41) Determine whether each function is even, odd, or neither. *Show your algebraic check.*

a) $f(x) = 2x^4 + 3x^2 + 4$

$f(-x) = 2(-x)^4 + 3(-x)^2 + 4$

$f(-x) = 2x^4 + 3x^2 + 4$

$\therefore$ even

b) $g(x) = 3x^5 + 2x^3 + x^2 + 1$

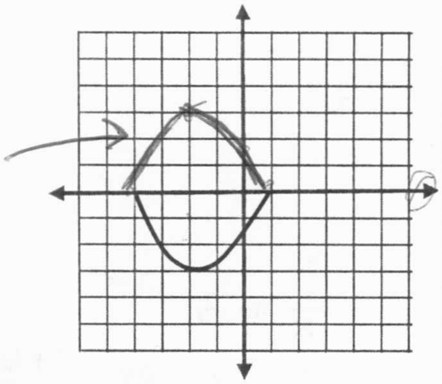
$g(-x) = 3(-x)^5 + (-x)^2 + 1$

$= -3x^5 + x^2 + 1$

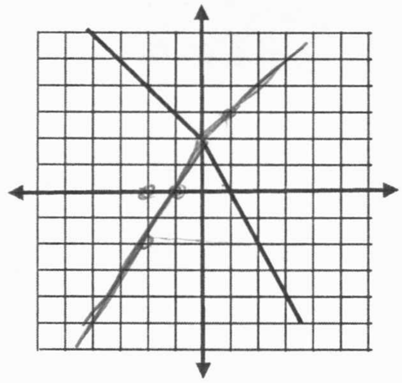
$\therefore$ neither

42) Graph the reflection of each function in the given line.

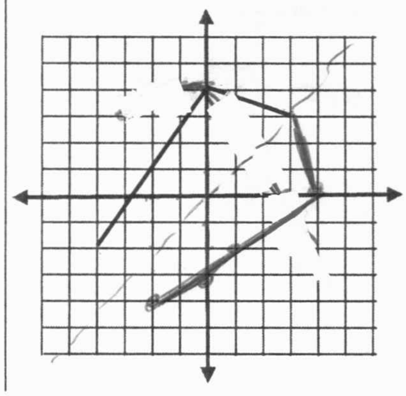
a) x-axis



b) y-axis



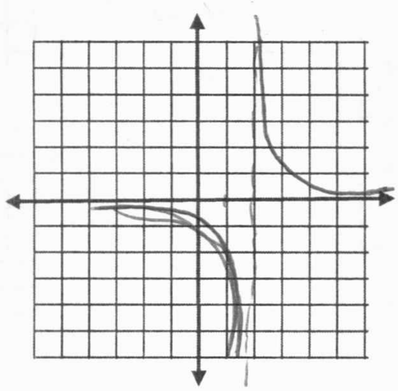
c) Line $y = x$



43) Graph each equation by determining the basic function and using transformations. *Write the "basic" function from which each is derived.*

a) $y = \dfrac{1}{x-2}$

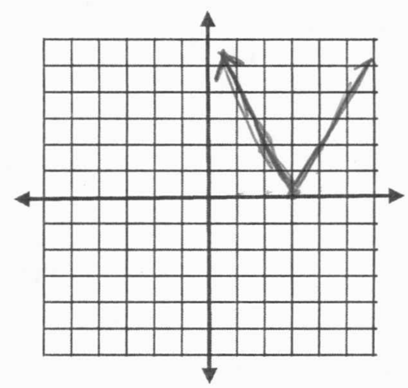
$y = \frac{1}{x}$
parent



$y = \underline{\frac{1}{x}}$

*This one won't be on your exam

b) $y = 2|x-3|$



$y = \underline{|x|}$

44) The height, s, in feet, of a ball thrown into the air is given by $s = -16t^2 + 96t + 112$, where t is the time in seconds. Find the maximum height and the time the ball takes to reach this height.

2 ways... ① Max on calculator
(3, 256)

maximum height = 256 ft

time to reach max height = 3 sec

② vertex form

$= -16t^2 + 96t \underline{\quad} + 112$

$= -16(t^2 - 6t \underline{+9}) + 112 \underline{+144}$

$y = -16(t-3)^2 + 256$

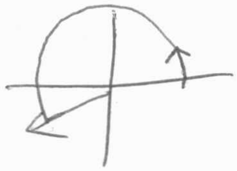
$y - 256 = -16(t-3)^2$ v: (3, 256)

TRIGONOMETRY

45) For an $\frac{11\pi}{9}$ counterclockwise rotation, find the measure of the angle in degrees. ~~Round answer to three decimal places~~.

angle measure = 220

$\frac{11\pi}{9} \cdot \frac{180}{\pi} = 220^\circ$



46) Express 218° 18' 46" in decimal form. Round answer to nearest thousandth.

omit

omit

47) Express $\frac{-7\pi}{5}$ in degrees.

$-\frac{7\cancel{\pi}}{5} \cdot \frac{180}{\cancel{\pi}} = -252$

-252°

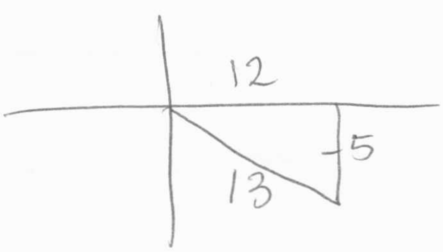
48) Express 318° in radians. Leave answer in terms of π.

$318 \cdot \frac{\pi}{180}$

$\frac{53\pi}{3}$

49) If $\cos\theta = \frac{12}{13}$, and θ is in Quadrant IV, determine the *exact* value of $\sin\theta$.

$\sin\theta = -\frac{5}{13}$

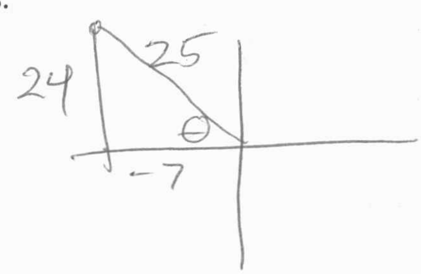


50) The terminal side of an angle θ in standard position passes through the point $(-7, 24)$. Determine the exact values of the six trigonometric functions.

$\sin\theta = \frac{24}{25}$ $\csc\theta = \frac{25}{24}$

$\cos\theta = -7/25$ $\sec\theta = -25/7$

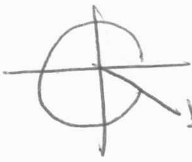
$\tan\theta = \frac{-24}{7}$ $\cot\theta = -7/24$



51) Determine the *exact* value of $\tan 690°$.

$\boxed{-\frac{\sqrt{3}}{3}}$

$$\begin{array}{r} 690 \\ -360 \\ \hline 330° \end{array}$$



$\frac{11\pi}{6}\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\tan = \dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

52) Evaluate $\sin 62.2°$ to four decimal places.

.8846

Calculater.
degree mode.

~~53)~~ If $0° \le \theta < 360°$ and $\cot\theta = -1.4176$, determine θ to the nearest tenth of a degree.

~~-.01~~

omit

For each function determine the amplitude, period, phase shift, vertical shift, and the equations for two vertical asymptotes, if applicable.

Function	Amp	Period	Phase shift	Vertical Shift	Domain	Range
54) $f(x) = 3\cos\frac{\theta}{4}$	3	8π	none	none	$(-\infty, \infty)$	$[-1, 1]$
55) $g(x) = \sin(3\theta - \pi) - 1$ $3(\theta - \frac{\pi}{3})$	1	$\frac{2\pi}{3}$	Right $\frac{\pi}{3}$	down 1	$(-\infty, \infty)$	$[-2, 0]$
56) $h(x) = \tan\left(2\theta - \frac{\pi}{3}\right) + 2$ $2(\theta - \frac{\pi}{6})$	none	$\frac{\pi}{2}$	Right $\frac{\pi}{6}$	up 2	all real #s except asymptotes $x \neq -\frac{1}{2}, \frac{3}{2}, \frac{7}{2}, \ldots$	$(-\infty, \infty)$

$\boxed{x \neq -\frac{1}{2} + 2n}$

57) Write a sine function with an amplitude of 3, with a period of $\frac{\pi}{2}$, and with a phase shift of $\frac{\pi}{6}$

$y =$ $3\sin 4\left(x-\frac{\pi}{6}\right)$

or

$y=3\sin\left(4x-\frac{2\pi}{3}\right)$

58) Write a tangent function with a vertical stretch of 3, a period of 2π, and a phase shift of $\frac{-\pi}{4}$.

$\frac{\pi}{b}=2\pi$

$b=\frac{1}{2}$

$y =$ $3\tan\frac{1}{2}\left(x+\frac{\pi}{4}\right)$

or

$y=3\tan\left(\frac{1}{2}x+\frac{\pi}{8}\right)$

59) Using your knowledge of the Unit Circle, find the exact values:

a) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ b) $\tan\left(-\frac{2\pi}{3}\right) = +\sqrt{3}$ c) $\csc\frac{7\pi}{4} = -\sqrt{2}$ d) $\cot\frac{13\pi}{6} = \sqrt{3}$

$\sin\frac{7\pi}{4}$ $\frac{1}{-\frac{\sqrt{2}}{2}}$ $\frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}}$ $\frac{\pi}{6}\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

e) $\sec\left(-\frac{9\pi}{2}\right) =$ $\cos$ $\boxed{\text{undefined}}$ $(0,-1)$ f) $\cos\left(-\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ g) $\sin\left(-\frac{15\pi}{4}\right) = \frac{\sqrt{2}}{2}$ h) $\tan\left(\frac{17\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

60) Given a point on the terminal side of an angle, find the exact value of the requested trig function.

a) $P\left(-8,\sqrt{17}\right)$

$\sin\theta = \frac{\sqrt{17}}{9}$

9 $\sqrt{17}$ -8

b) $P(3,-5)$

$\cos\theta = \frac{3}{\sqrt{34}} = \boxed{\frac{3\sqrt{34}}{34}}$

3 θ 4 -5 $\sqrt{34}$

c) $P\left(\frac{1}{3},\frac{1}{4}\right)$

$\tan\theta = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{1}{4}\cdot\frac{3}{1} = \frac{3}{4}$

$\frac{5}{12}$ $\frac{1}{4}$ $\frac{1}{3}$

61) You are riding a bicycle with wheels that have a radius of 25 inches. If you are traveling at 18 miles per hour, what is the rotational speed of the tires in revolutions per second? What is the angular speed of the wheels in radians per second?

$\frac{18\,\cancel{mi}}{1\,\cancel{hr}}\cdot\frac{1\,\cancel{hr}}{3600\,sec}\cdot\frac{5280\,\cancel{ft}}{1\,\cancel{mi}}\cdot\frac{12\,\cancel{in}}{1\,\cancel{ft}}\cdot\frac{1\,rev}{2\pi(25\,\cancel{in})} = \boxed{\frac{2.02\,\cancel{rev}}{sec}}\cdot\frac{2\pi\,rad}{1\,\cancel{rev}} = \boxed{12.69\,rad/sec}$